ERRORS AND MISCONCEPTIONS IN SOLVING QUADRATIC EQUATIONS BY COMPLETING A SQUARE

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Abstract

This paper is based on the qualitative study that was conducted in five South African schools of Limpopo Province in Capricorn district. The main aim was to diagnose errors learners made in solving quadratic equations by completing a square and found the reasons why those errors occurred. The results revealed that learners in those schools experienced problems in solving quadratic equations by completing a square. Findings revealed that the teaching approaches used by teachers contributed towards learners' errors and misconceptions they possessed in solving quadratic equations by completing a square. Further, findings showed that learners were not given an opportunity to discuss the concepts and they did not have enough time to practice their work. Other reasons found in the five schools are – learners are afraid to be mocked at by their peers during the lessons, learners themselves not committed to their school work, some have parental role to play at home, and some teachers are not committed as they come to class late and not controlling their work. The recommendation is that in identifying learners' errors and misconceptions teachers should also diagnose those errors in order to understand learners' challenges.

INTRODUCTION

Mathematics resolves around conceptual dissemination and skills attainment and enhancement. This enables learners to acquire new mathematical skills and knowledge or concept to consolidate the old ones (Luneta, 2008). Teachers in schools use different teaching methods such as group work, exposition and explanatory, problem solving, practical work, direct instruction; this may be the application of constructivist approach in teaching and learning of mathematics. The key element in the approaches is for learners to use mathematical skills and knowledge confidently in solving mathematical problems. Learners are evaluated thereof to on their understanding of the concepts in mathematics and the quality of teaching, and this helps to reveal learners' errors and misconceptions (Borasi, 1994; Riccomini, 2005).

Mathematics teachers did not treat learners errors made in solving mathematical problems seriously. Instead, those errors were ignored, under the guise that mathematics was being undertaken by intelligent learners and those whose performance is on average were reflective of 'general streams'. This issue of ignoring the errors and misconceptions of learners in mathematics also occurred in the five schools I used in the study. Teachers did not have courage to investigate learners' problems experienced in solving mathematical problems, until I had a casual discussion on performance of learners' with two teachers from different schools in the vicinity.

Arising from this casual discussion, it was concluded that there was a need to analyse concepts that seemed to pose challenges. I started by looking at simple concept like grade 11 quadratic equations and teachers identified to me that learners had a serious problems in solving quadratic equations by completing a square. The poor performance of learners motivated me to think more about the types of errors learners displayed and misconceptions they possessed. In subsequent discussion with the teachers, we decided to give learners a diagnostic test to understand those errors and misconceptions in solving quadratic equations by completing a square.

This paper describes the types of errors and misconceptions the grade 11 mathematics learners grapple with and why those errors displayed and how they arrived at such misconceptions in solving quadratic equations by completing a square. The study also discusses in conceptualisation and interpretation which lead learners to wrong answers. It also aimed to assist teachers to be more sensitive about learners' errors and misconceptions in mathematics. This study will also assist learners to realise the types of errors they display in solving quadratic equations by completing a square and other mathematical concepts. They gained conceptual and procedural knowledge of the quadratic equations by completing a square.

LITERATURE REVIEW

Definition of errors and misconceptions

Hansen et al., 2006: 15-16 define errors and misconceptions 'errors are mistakes learners make when solving problems that may be caused by carelessness, misinterpretation of symbols or text; lack relevant experience or knowledge related to that mathematical topic, learning objective, concept; lack of awareness or inability to check the answer given; or the results of misconceptions. Misconception is the misapplication of a rule, an over- or under-generalisation, or alternative conception of the situation'. Error discourse has since been discussed long time ago in the 17th century for instance, the work of Bacon in 1620 and also Pierce in 1887 cited in Luneta 2008. Learners make errors in some situations without realising them and if those errors are not recognised or dealt with, those errors can lead to more errors (Pickthorne, 1983).

Pickthorne (1983: 1285) also asserts that too often lecturers appear not fully realise the extend or the nature of their learners' confusions. They are unable to discover the reasons why those errors are made by the learners.

When we diagnose mathematical difficulties, we determine the areas of weaknesses a learner has, we study a specific errors the learner is frequently making, and we attempt to explain why those errors are being made (Troutsman & Alberto, 1982). Most of the teachers identify learners' errors but rarely diagnose them (Luneta, 2008). When analysing and diagnosing errors leaners displayed, we identify the root cause of those errors and how best they can be corrected in order to benefit both learners and teachers. Learners will not commit same errors and teachers will not teach same concept to the same learner. Errors and misconceptions in mathematics are mostly influencing learners' learning of mathematics and hence quadratic equations by completing a square, my focus area. Teachers' mistreatment of errors and misconceptions has exacerbated their effects and has retarded learning (Pickthorne, 1983).

Quadratic equations by completing a square

This method of solving quadratic equations by completing a square is helpful as it was appropriately applied in finding the solution to the equations; learners were alerted to use this method appropriately to provide them with the correct answers. In completing a square according to Laridon et al. (2010), learners should ensure that the coefficient of x^2 is 1 and if it greater or less than 1, they should divide by that coefficient, $ax^2 + bx + c = 0$. Learners should divide by a throughout before they could find the additive inverse of c both sides to have $x^2 + \frac{b}{a} = -\frac{c}{a}$. Learners can then add half the coefficient of x both sides before the equation could be factorised, $x^2 + \frac{bx}{a} + \frac{b^2}{4a^2} = -\frac{c}{a} + \frac{b^2}{4a^2}$. The equation can then be factorised on the left hand side and be simplified on the right hand side (Pretorias et al., 2006).

Conceptual and procedural knowledge

Conceptual knowledge is knowledge about the facts, concepts or principles upon which something is based (Microsoft Office Dictionary, 2001). Herbert and Lefevre (1986: 3-4) define conceptual knowledge as 'knowledge that is rich in relationship, that can be thought of as a connected web of knowledge, a network in which linking relationships are as prominent as the discrete pieces of information'. Such knowledge is described as that which is interconnected through relationships at various levels of abstraction (Confrey, 1990). It plays a more important role in the learning of mathematics than procedural one. It is essential for learners to have conceptual understanding, as in the absence of which they will ineffectively indulge in problem solving and follow wrong procedures to solve them (Center for Develop Enterprise, 2007).

Zemelman, Daniels, & Hyde (1998: 89-90), state that 'without true understanding of the underlying concepts, serious problem [guaranteed] with learning other concepts'. Focusing on understanding mathematical ideas makes students 'far more likely to study mathematics voluntarily and acquire further skills as they are needed.'

Teachers should know their learners' mathematical thinking to be able to structure their teaching of new ideas to work with or correct those ways of thinking, thereby preventing learners from making errors (Sorensen, 2003). The way learners think about a concept depends on the cognitive structures learners have developed previously (Battista, 2001). Battista (ibid) also indicates that if learners cannot develop concepts by themselves, they will have a narrow understanding of those specific concepts, and will not be able to engage themselves in problem solving. Learners who do not have background knowledge in mathematics usually display numerous errors in solving mathematical problems, and this therefore results in most of learners grappling with quadratic equations by completing a square. Conceptual knowledge works hand in hand with procedural knowledge.

Procedural knowledge should be taught in mathematics to reinforce understanding of mathematical concepts. Procedural knowledge is the ability of learners to use the relevant procedures in solving mathematical problems by following the rules, methods and procedures in different representations (Kanyaliglue, Ipek, & Isik, 2003; Hiebert & Lefevre, 1986). Procedural knowledge is a particular type that learners display in solving problems and also in adhering to certain instructions when completing different tasks (Hiebert & Lefevre, 1986). Luneta (2008) asserts that procedural knowledge is specific to a particular task, and this implies that some procedures are not appropriate to solve certain mathematical problems. Learners may grasp relevant procedures but fail to use them correctly in solving mathematical problem (Siegler, 2003). Learners who lack this understanding are frequently using wrong procedures and this generates systematic patterns of errors in solving problems. Accordingly, teachers should not focus only on fact errors, but also on basic errors, especially when learners are making the same procedural errors (Gernett, 1992; National Research Council, 2002; Stein et al., 1997).

Riccomini (2005) explains instructional focus on facts as being problematic to teachers teaching parts of concepts or parts of procedural steps because teachers are trained to teach mathematics in terms of general concepts, and therefore this helps in addressing the problem of learners solving quadratic equations by completing a square. Riccomini also states that the teachers' ability to recognise error patterns can be improved and that it might be possible to plan instructions that can help to alleviate problematic patterns in this concept. Measures to this effect might be pre-service programmes, professional development opportunities in mathematics, refining curriculum materials, and continued research in mathematics for learners with disabilities.

There is a relationship between procedural knowledge and conceptual knowledge. The correlation is shown when a learner is able to execute the procedures correctly and this displays a good grasp of conceptual knowledge (Siegler, 2003). If a learner has both procedural and conceptual knowledge, s/he can solve more complex problems of the same concept and this will help grade 11 learners solve quadratic equations by completing a square. Learners with conceptual understanding produce substantial gain in in both kinds of knowledge, but those with procedural understanding produce substantial gain in procedural knowledge but less in conceptual knowledge, which will ultimately impede learners' growth in mathematics. Nesher (1986) supports the view that if a learner can only be shown procedures of solving a particular problem without understanding the concept, it is very unlikely that such a learner would be in a position to solve more complex problems independently. If problems are difficult to solve, then learned procedures may not help and will need a learner to have conceptual knowledge to solve them Nesher (1986). Conceptual knowledge also provides and constraints those procedures to be followed in solving mathematical problems (Confrey, 1990).

RESEARCH DESIGN

The study used a diagnostic test followed by focus group interviews to understand the reasons behind the errors and misconceptions learners had in solving quadratic equation by completing a square. It was conducted under qualitative research approach in which the information collected was analysed through description and not statistically (Rule & John, 2011). The study had been conducted in five schools of same circuit which comprises of 11 secondary schools. The focus of the study was on diagnosing errors and misconceptions learners experienced in solving grade 11 quadratic equations by completing a square. Sixty five scripts were sampled from five schools, in which 15 scripts were randomly sampled from each school and 10 learners participated in the focus group interviews which gave a total of 50 learners. Each group of learners were represented by five female learners and five male learners to observe gender equality. The reason for these participants was to find out the root causes and reason why those errors and misconceptions occurred in solving those equations by completing a square.

Instrument

The instruments used in this study were the diagnostic test and interview schedule. The diagnostic test assisted in identifying and diagnosing the types of errors and misconceptions learners had in the five schools in solving quadratic equations by completing a square (Salvia & Ysseldykes, 2004). The interview schedule helped me in understand the reasons why leaners displayed those errors and misconceptions possessed.

The collected data through diagnostic test and focus group interviews attempted to respond to the following objectives of the study:

1. To identify the types of errors learners displayed in solving quadratic equations by completing a square and the misconceptions possessed and analyse them.
2. To diagnose those errors and misconceptions learners had and how well they could be addressed.

FINDINGS AND DISCUSSIONS

The findings and discussions are divided into two components in which the first one dwells on the types of errors and misconceptions identified in solving quadratic equations by completing a square. The second component dwells on the reasons why those errors and misconceptions occurred by using a focus group interview. Furthermore, some reasons why those errors were displayed by those learners were found and were benchmarked against the data collected from the diagnostic test.

Component 1: Types of errors and misconception

The first component of the analysis of learners' scripts in solving quadratic equations by completing a square was to identify the errors and misconceptions learners made. The collected data was analysed by identifying the common errors learners displayed in solving equations by completing a square. Those common errors were characterised by conceptual errors and procedural errors. Most of the common errors found were dividing by the coefficient of x^2 if the equation was greater than 1 or less than zero. Moreover, some of the common errors were the ones of simplifying $\left(2 \times \frac{1}{2}\right)^2$ from the equation, $x^2 - 2x - 1 = 0$. Most of them got the answer as 2 and pursued them by the focus groups to understand why they solved the problem in that way. Other learners did not find the additive inverse of a constant -1 before completing a square, which was also wrong for them to solve the equation in that fashion. More errors were found when some of them were failed to factorise the equation after completing a square which revealed that learners lacked knowledge of factorisation. Others rewritten their equations in the form of $ax^2 + bx + c = 0$ and then factorised instead of factorising the equation without writing it in standard form. Some of them found the additive inverse of the equation but only completed a square on the left hand side and failed to do it on the right hand side. Another common error was that some of the learners had failed completely to attempt to complete a square in solving quadratic equations, such as dividing the coefficient of x^2 if was greater or less than 1. Laridon et al., (2011) advise that when adding half the coefficient of x, learners should ensure that the coefficient of x^2 is 1. They also further stated half the square of the coefficient of x should be added both on the left hand side and the right hand side. This view was supported by Zemelman et al., (1998) that learners without true understanding of the underlying concepts guarantee serious problems in learning other concepts. This is what happened to these learners as this reveal that most of

them were unable to solve quadratic equations by finding factors and this had continued on to this one of completing a square. Siegler (2003) also asserted that a learner without a good understanding of a concept result in using procedures of solving problems inappropriately. Most of these learners did not know how to solve the equations and showed that specific knowledge of procedures was not acquired by those learners. Luneta (2008) supports that procedural knowledge is specific to a particular task. If a learner does not understand the concept, it would be difficult for him/her to articulate the procedures to solve problems based on that particular concept (Battista, 2001).

Component 2: Diagnosing errors and misconceptions

In diagnosing errors and misconceptions, I understood the reasons why the leaners had those errors and misconceptions in solving quadratic equations by completing a square. The reasons behind those errors and misconceptions were found by using focus group interviews. In focus groups, some indicated that this concept is challenging to them as they compared with factorisation and using quadratic formula. In the discussion, learners revealed that some of their teachers don't give them an opportunity to participate and only the fast learners were always given a platform. In some schools learners were unable to participate as their teachers praised those who gave correct answers. Those learners were again unable to participate as they were afraid of being mocked at by their fellow learners. Learners also blamed themselves for not being committed to their school work due to their laziness, some mentioned that they had parental role to play at their homes. It was also revealed that the approaches teachers used to teach mathematical concepts were difficult to understand. They said some of the teachers were fast and don't give learners chance to engage with them. It was also indicated that other teachers were not committed to their work, like coming to class late and not controlling their work. From the focus group interviews, most of the learners were not given an opportunity to discuss the concepts and some indicated that they did not have enough time to practice their work.

CONCLUSION REMARK

It is imperative for teachers to teach mathematics using learners' errors and misconceptions as this will guide them on what learners grapple with. They will be able to use multiple strategies or approaches to teach mathematical concepts to cater all the learners including the slow ones. In this study learners were unable to divide by the coefficient of x^2 if the equation was greater than 1 and less than zero, multiplying by half the coefficient of x and also additive inverses of any constant given. Some reasons which contributed to these difficulties were: learners' laziness, learners' lack of participation, teachers' teaching participation, teachers' commitments and conduct towards learners; and peers' behaviour in the classroom. Teachers are not supposed to identify learners' errors and misconceptions but they should also diagnose those errors in order to understand learners' challenges.

REFERENCES

Battista, M. T. (2001). A Research-based Perspective on Teaching School Geometry. In J.   Brophy (Ed.). Subject-specific instructional methods and activities, advances in   research on teaching. (Vol. 8), (pp. 145-185). Elsevier Science.

Borasi, P. (1994). The New Organ or true directions concerning the interpretation of nature. *www.constitution.org/bacon/nov org.htm*. Also in the Great Instraution/Novum   organum. Modern Library. 1955 (1620).

Centre for Development Enterprise (CDE, 2007). *Doubling for Growth: Addressing the maths   and science challenges in South Africa.* Centre for Development Enterprise.

Confrey, J. (1990). A Review of the Research on Student Conceptions in Mathematics,   Science and Programming. In C. B, Carden (Ed), *Review of Research in Education.*   Washington: American Educational Research Association.

Garnett, K. (1992). Developing Fluency with Basic Number Facts: Interventions for Students   with Learning disabilities. *Learning Disability Research and Practice*, 7, 210-216.

Hiebert, J., & Lefevre, P. (1986). Conceptual and Procedural Knowledge in Mathematics: An   Introductory Analysis. In J. Hiebert (Ed), Conceptual and Procedural Knowledge: The   Case of Mathematics. pp. 1-27. Hillsadale, NJ: Erlbaum.

Kanyalioglu, A. C., Ipek, A. S., & Isik, A. (2003). On the teaching linear algebra at University   level: The role of visualising in the teaching vector space. Journal of Korea Society of Mathematics Education Series D. *Research in Mathematics Education*, 7(1), 59-67.

Laridon, P., Barnes, H., Jawrek, A., Pike, M., Myburgh, M., Houghton, R. R., Scheber, J.,   Segabi, M., & Wilson, H. (2010). *Classroom Mathematics Grade 11*. Sandton: Heinemann Publishers.

Luneta, K. (2008). The professional development model of evaluating and enhancing. *In the International council of the Education of the Teachers*: Year book 2008.

Microsoft Office Dictionary 2001

National Research Council (2002). Helping children learn mathematics. *Mathematics   Learning Study committee,* J. Kilpatrick & J. Swafford, (Eds), Centre for Education.   Division of Behavioral and Social Sciences and Education. Washington, DC: National   Academy Press.

Nesher, P. (1986). Are Mathematical Understanding and Algorithmic Performance Related?   *For the Learning of Mathematics, 6(3), 2-9.*

Pickthorne, B. (1983). Error factors: A missing link between cognitive science and classroom   practice. Instruction Science 11: 281-312.

Pretorius, J., Potgieter, R., & Ladewig, W. (2006). Mathematics Plus, Grade 11. Cape Town,   Oxford University Press.

Riccomini, P. J. (2005). Identification and remediation of systematic error patterns in   subtraction. Learning disabilityQuarterly, 28(3), 233-242.

Salvia, J. & Ysseldykes, J. E. (2004). Assessment (9th ed) Boston: Houghton Mufflin   Company.

Siegler, R. S. (2003). *Implantation of Cognitive Science Research for Mathematics   Education.* Carnegia Mellon University. In Kilpatrick.

Sorensen, A. (2003). Effective Teaching high School Mathematics.

Stein, M., Silbert, J. & Carnine, D. (1997). *Designing effective mathematics instruction: a direct instructional approach (3rd ed.)*, Columbus OH: Merril.

Troutsman, A.C. & Alberto, P. A. (1982). Applied Behavior analysis for Teachers: *Influencing student Performance*. Columbus, OH: Mrril Publishing.

Zemelman, S., Daniels, H., Hyde, A. (1998). Best practice: New standards for teaching and learning in American's schools (2nd ed.). Portsmouth, NH: Heinemann.